L.A. GRIGORYAN, V.A. SHAKHBAZIAN

EMC-EFFECT AND QCD EVOLUTION OF THE THREEQUARK NUCLEON PICTURE

ЦНИИАтоминформ
ЕРЕВАН-1985
1. Introduction

The experiments on the deep inelastic muon scattering on deuterium [1] and Ferrum [2] nuclei have shown that the bound and free nucleon structure functions do not coincide with each other. The consequence of this fact is the nontrivial behaviour of the ratio of the nuclear structure function in Ferrum to the same one in deuterium as a function of the Bjorken variable $x$ [3]. The effect does not disappear when other nuclei are considered and when the muon beam is replaced by an electron one. This phenomenon, called the EMC-effect, is very interesting, since it reveals new possibilities for the investigation of nuclear structure on the quark level [4].

To explain this phenomenon, a great number of theoretical works have been done [5-18], in which two main ideas are developed.

The first one is the partial deconfinement in nuclei, leading to extension of the bound nucleon effective dimension.

The second one is to take into account the nonnucleon states in the nuclear wave functions ($\Xi$-mesons, $\Delta$-isobar, and so on).

Some authors try to explain the behaviour of structure functions ratio $R = \frac{(\frac{dA}{dx}) F_2^{(Fe)}}{(\frac{dA}{dx}) F_2^{(2H)}}$ in the regions of large or small $x$ by means of the first or the second assumptions, respectively.
ely, considering them mutually complementing. Most important to us is the verification of partial deconfinement of quarks in nuclei i.e. the verification of the colour objects (quarks and gluons) exchange possibility between the nucleons in a nucleus. This must effectively lead to extension of nucleon dimension in nuclei. While studying this problem one must bear in mind, that in the deep inelastic region hadrons (including also nucleons) are presented as groups of partons (current quarks and gluons), characterised by their distributions which, however, do not contain an explicit dependence on the hadron dimension. Therefore, to describe the nucleon swelling one must either phenomenologically introduce the dependence on nucleon dimension into the scaling variable (see, e.g. Ref.[7]) or take it indirectly into account by investigating its dependence on the normalization momentum [12-14]. In the latter case the obviousness of consideration is somewhat lost and the choice of boundary conditions seems to be a little arbitrary.

The derivation of structure functions in the deep inelastic region (of large $Q^2$) by means of the QCD approach, using the Altarelli-Parisi evolution equations for the distributions of quarks (and gluons), is more consistent. Then it appears possible to use the constituent quark distributions as boundary conditions at small $Q^2$. The constituent quark distributions contain the dependence on the nucleon dimension, which will already appear in the final solution of the equations. Such an approach, developed, in particular, in the work of F. Martin [19] (see also Refs [20-23]), gives the possibility to take consequently into account the dependence on the nucleon radius and, thus, the effect of nucleon swelling in a nucleus.

In this paper an attempt is made to investigate the EMC-effect just in such an approach.

For this, the structure function of bound nucleon is obtained from that of free nucleon simply by changing the radius, which is found by fitting the theoretical expression to the EMC data. It should be noted that to exhibit in the pure form the effect of nucleon swelling in a nucleus we confined ourselves by the $X$ variable region of $0.25 \leq X \leq 0.65$. This is done to be distracted for the present from the contribution of the nonnucleon components (pions, $\Delta$-isobars and so on) which are essential at $X < 0.25$, and from the corrections of Fermi motion which are essential at $X > 0.65$ (it should be noted, however, that the contribution of both kinds of corrections can be taken into account in the present approach, which is to be done later). As it is shown in this work, the growing of the nucleon radius in Ferrum nucleus by 10% leads to a good agreement with the experimental data and is consistent with the values from the other works [7,12,14].

2. Description of the model

Following to Ref.[19], let us briefly describe the results of the QCD modifications of the threequark nucleon picture. The valence quark, quark-­antiquark sea and gluon distributions may be consequently written in the form of

$$ q_{vu}(x,Q^2) = \int_x^1 \frac{dy}{y} q_{v}(\frac{x}{y}) \tilde{F}_{vu}(y,t),$$

$$ q_{is}(x,Q^2) = \int_x^1 \frac{dy}{y} q_{v}(\frac{x}{y}) \tilde{F}_{is}(y,t),$$

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\[
G(x, Q^2) = \sum_{x} \frac{d^2y}{y^2} \cdot q_{V}(\frac{x}{y}) \cdot F_{QV}(y, t),
\]

where \( q_{V}(x) \) is the structure quark (valon) distribution in a nucleon which we identify at small \( Q^2 \) with the distribution \( q_{V}(x, Q^2) \). \( q_{V}(x) \) are the probability densities to reveal inside a structure quark, probing by the transfer momentum \( Q^2 \), the parton of \( i \)-th kind \( (i = u, d, g) \) which takes away the \( y \) part of the structure quark momentum. The probability densities are connected with the functions \( F_{u} \), \( F_{d} \) and \( F_{g} \), obtained in Ref.[19], by the inverse Mellin transformations

\[
\tilde{F}_{uv}(y, t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn \cdot n^{-\frac{1}{2}} \cdot \tilde{F}_{uv}(y, n). \tag{4}
\]

Since we shall use only \( F_{uv}(n, t) \), let us write down its explicit form [19]

\[
F_{uv}(n, t) = e^{t A_{uv}(n)}, \tag{5}
\]

where

\[
A_{uv}(n) = \frac{3}{4} - \frac{1}{2n} - \frac{1}{2(n+1)} - \Psi(n+1) - C,
\]

\[
t = \frac{16}{33 - 2\pi} \cdot \ln \left( \frac{K}{\alpha(Q^2)} \right), \quad C = 0.577,
\]

\[
\Psi(x) = \frac{d}{dx} \cdot \ln \Gamma(x)
\]

is the psi-function, \( K \) is the fitting parameter. Note also, that \( F_{uv}(n, t) \) is involved in the solution for the valence quark distribution momenta

\[
M_{q_{V}}(n, Q^2) = M_{q_{V}}(n, Q^2) \cdot F_{QV}(n, t), \tag{6}
\]

which is obtained by means of the standard procedure from the Altarelli-Parisi equation

\[
\frac{d^2 q_{V}(x, Q^2)}{dt} = \frac{\alpha(Q^2)}{4\pi} \int_{x}^{1} \frac{d^2y}{y^2} \cdot \tilde{F}_{q_{V}}(x, Q^2), \tag{7}
\]

where

\[
\tilde{F}_{q_{V}}(Z) = \left\{ \begin{array}{l}
2 (1-Z) - (1+Z) + \frac{3}{2} \delta(Z) \\
(\int_{0}^{1} \frac{dZ}{(1-Z)^2} \equiv \left( \int_{0}^{1} \frac{f(\tilde{Z})}{1-\tilde{Z}} \right) \right. \right.
\]

\[
\left( \int_{0}^{1} \frac{f(\tilde{Z})}{(1-\tilde{Z})^2} \right) \equiv \left( \int_{0}^{1} \frac{f(\tilde{Z}) - f(1)}{1-\tilde{Z}} \right) \right) \right).
\tag{8}
\]

(For the details of derivation of all three distributions see formulae (1)-(8) in Ref.[19]).

The key point of this consideration is that in a sufficiently extensive region of small transfer momenta \( Q_{V} \), the nucleon can be represented as a combination of three valence quarks or valons. The part of sea quarks and gluons being equal to zero. Therefore, following to [19], the boundary conditions are taken in the form of

\[
x \cdot q_{V}(x) = 0,
\]

\[
x \cdot q_{s}(x) = 0,
\]

and

\[
x \cdot q_{g}(x) = x \cdot (u_{V}(x) + d_{V}(x)) =
\]

\[
= N_{1} x \exp \left[ -\frac{3}{2} \cdot R_{N}^{2} \cdot m_{N}^{2} (x - \frac{1}{2}) \right] + N_{2} \sqrt{x} (1-x)^{3},
\]

or

\[
x q_{u}(x) = \frac{N_{1}'}{[(x - \frac{1}{2})^{2} + \beta N_{2}^{2} m_{N}^{2}]} + N_{2} \sqrt{x} (1-x)^{3},
\]

\[
7
\]
The first term in Eq. (9) corresponds to the oscillator model [21] and to that in Eq. (10) to the exchange model of vector gluon. The term $\sqrt{1-x}^3$ in the right hand sides of Eqs (9) and (10) is introduced to keep the law of conservation of momentum and the number of valence quarks.

\[
\int_0^1 x q_v(x) dx = 1, \quad (11)
\]

\[
\int_0^1 q_v(x) dx = 3. \quad (12)
\]

It is important for us, that the distributions (9) and (10) both contain an explicit dependence on the nucleon radius. The fitting parameter $K$ is determined by experiment.

\[
\int_0^1 F_{2e}^p(x) dx \approx 0.18 \quad \text{at } Q^2 = 4\text{GeV}^2. \quad (13)
\]

The numerical value of $K$ is

\[
K \approx 0.7. \quad (14)
\]

If we equated $K = \delta^2(Q^2)$, we should have obtained that

\[
\delta^2(Q^2) \approx 0.7, \quad \text{and it could be possible to determine } Q_0. \quad \text{However, one must be careful here, since } \delta^2(Q^2) \approx 1, \text{ then the contribution of higher approximations might be important, their sum being equal to } K = \delta^2(Q^2) \text{ [19], which justifies the choice of such boundary conditions.}
\]

When deriving the valence quark distribution $q_v(x, Q^2)$, the inverse Mellin transformation must be satisfied to find $\tilde{F}_{uv}(x, t)$. For this, one may use the simple way in Ref. [25], to obtain the relation between the momenta of $\tilde{F}_{uv}(x, t)$ and $\tilde{F}_{uv}(x, t)$ itself with the help of Legendre polynomials.

The point is, that if $\tilde{F}_{uv}(x, t)$ is assumed to be equal to zero in the region $-1 < x < 0$, then the initial expression for momenta

\[
K_{uv}(n, t) = \int_0^1 dx (x \tilde{F}_{uv}(x, t)) x^{-2} \quad (15)
\]

is compatible with transformation

\[
\tilde{K}_{uv}(n, t) = \int_0^1 P_n(x)(x \tilde{F}_{uv}(x, t)) dx = \int_0^1 P_n(x)(x \tilde{F}_{uv}(x, t)) dx, \quad (16)
\]

where $P_n(x)$ is the Legendre polynomial.

It is obvious that $\tilde{K}_{uv}(n, t)$ are the linear combinations of $K_{uv}(n, t)$, and then, determining $x \tilde{F}_{uv}(x, t)$ from Eq. (17), we obtain

\[
x \tilde{F}_{uv}(x, t) = \sum_{n=0}^{\infty} \frac{2n+1}{2} P_n(x) \tilde{K}_{uv}(n, t). \quad (18)
\]

Thus, knowing the momenta $K_{uv}(n, t)$, it is possible to construct $\tilde{K}_{uv}(n, t)$, and with their help, according to Eq. (18), to determine $x \tilde{F}_{uv}(x, t)$ as well. It is shown in Ref. [25] that this procedure has a high accuracy. In the results of calculations presented later, as far as 12 terms of the series are considered, since the contributions of higher momenta are negligible.*

* We have described this technical point in detail, since the use of the Legendre polynomials for obtaining the inverse Mellin transformation is considerably simple as compared with the usual methods [19, 22].
J. Results of calculation and discussion

The region of $0.25 \leq X \leq 0.65$ is considered in the work. As it is noted in the introduction, the choice of the lower limit is owing to the fact that during the calculation the contributions of the nuclear wave function nonnucleon components (pions, isobars and so on) and those of sea quarks, essential at $X < 0.25$, have not been taken into account. And the choice of the upper limit is owing to the fact that the Fermi motion of nucleons in nucleus, essential at $X > 0.65$, has not been taken into account too. Deuterium and Ferrum nuclei were used as targets. The deuteron is an isoscalar, and there is a little neutron excess in the Ferrum which is inessential in the numerical calculations (its addition is $\sim 1.5\%$). The structure function per nucleon for deep inelastic scattering on isoscalar targets (neglecting the contribution of the sea quarks) has the form

$$
\frac{1}{A} F^D_{2\nu}(x, Q^2) = \frac{5}{16} x q_{F\nu}(x, Q^2) \quad (19)
$$

The nucleon structure functions in deuterium and Ferrum, depending on $Q^2$, were calculated at values of $X$ equal to 0.25, 0.35, 0.45, 0.55 and 0.65. The theoretical values were compared with the experimental data from the works in Refs [1,2,4]. The result of the comparison is presented in Figs 1 and 2. The parameter $\Lambda$ and the nucleon radius $R_N$ were fitted during the calculation. In the case with deuteron the values of $\Lambda = 0.12\text{GeV}$, $R_N = 0.48\text{fm}$ were obtained. $\chi^2$ per point appeared to be $\sim 1.5$ (only statistical errors were taken into account [1,4]). It should be noted that taking of the systematical errors into account too, even on the level of 20% of statistical errors, would make $\chi^2$ as low as 1. In the case with Ferrum we obtained $\Lambda = 0.12\text{GeV}$ and $R_N^{(\text{Fe})} = 0.53\text{fm}$. $\chi^2$ per point was obtained to be about 0.5 (both, statistical and systematical errors on the level of 7.5% were taken into account [2]).

Thus, the nucleon radius, fitted to the deuteron data, is, as expected, the same as that of the free nucleon [19]. In Ferrum an effective swelling of nucleon takes place with a radius increase of 10% in comparison with the free nucleon case. Further, in the works where the experimental data with relatively small $Q^2$ ($Q^2 \sim 10\text{GeV}^2$) were used, the value of $\Lambda$ was obtained to be about 0.5GeV. The modern experiment has promoted to the region of hundreds of GeV$^2$ ($Q^2 \sim 100\text{GeV}^2$). That is why the values of $\Lambda$ tend to drop down to 0.1-0.2GeV [28], and the value we obtained for $\Lambda$ agrees to this. The reason of this situation is the improvement of the applicability of the perturbative QCD, i.e. the decreasing of the role of the nonperturbative effects, when $Q^2$ is increasing.

The ratio of the averaged over $Q^2$ structure functions of Ferrum and deuteron, as a function of $X$ [3,4], is presented in Fig.3. The theoretical points, shown by crosses, are in quite a good agreement with the experiment. For comparison the theoretical calculation of the structure function ratio at $Q^2 = 50\text{GeV}^2$ is given. For completeness the comparison of the ratio $R$ with the experimental data of the E09-02 collaboration [4,5] at the values of $Q^2$ equal to $55\text{GeV}^2$, $65\text{GeV}^2$, $80\text{GeV}^2$, $105\text{GeV}^2$, $140\text{GeV}^2$ and $180\text{GeV}^2$ is represented in Fig.4. The agreement is rather good.
In conclusion we want to stress that the suggested here approach has definite advantages. In the framework of QCD it succeeded to include the parameters of the structure quark picture in the explanation of the EMC-effect. This allowed to observe the effect of nucleon swelling in nuclei in an explicit way. Further, in this approach it is quite possible to take the non-nucleon components of the nuclear wave function (pions, Δ-isobars and so on) into account by the corresponding choice of the boundary conditions at small $Q^2$, which will be the subject of a further consideration. The represented derivation shows also, that the valon distributions in nucleons and in nuclei are different, which, generally speaking, have not been yet taken into account [26,27].

The authors are thankful to prof. A.Ts. Amatuni for support and to the members of theoretical seminar of Yerevan Physics Institute for useful discussion.

Fig. 1 The structure function $\frac{1}{2} F_2^{(R)}(x,Q^2)$ for deuteron as a function of $Q^2$ at the fixed values of $x$, equal to $0.25; 0.35; 0.45; 0.55$ and $0.65$. The experimental points are taken from [1,4] for the energy of the incident muon of 280 GeV. The solid lines show the theoretical calculation according to the formula (19). Only the statistical errors are presented.
Fig. 2 The structure function $F_2^{(Fe)}(x, Q^2)$ for the Fe as a function of $Q^2$ at the fixed values of $X$, equal to 0.25; 0.35; 0.45; 0.55 and 0.65. The experimental data are taken from Ref. [2] for the energies of the incident muon equal to 120 GeV ($\oplus$), 250 GeV ($\oplus$) and 280 GeV ($\oplus$). Solid lines show the theoretical calculations according to the formula (19). Only the statistical errors are presented.

Fig. 3 The ratio $\bar{R} = \frac{(Y_0) F_2^{(c)}(x, \bar{Q}_R^2)}{(Y_0) F_2^{(o)}(x, \bar{Q}_R^2)}$ as a function of $X$. The experimental points (black circles with the errors) are the data of EMC [3,4]. The dashed line shows the theoretical calculation made for $Q^2 = 50$ GeV$^2$. Crosses are the theoretical values of the ratio $\bar{R}$, calculated for the mean values of $Q^2$ (their own for each $X$): $\bar{Q}_R^2 = \frac{1}{n_R} \sum_{i=1}^{n_R} Q_i^2$ for each $X$, $n_R$ is the number of the points for each $X$. 

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Fig. 4 The ratio $R = \frac{\frac{1}{2} f_{2}^{(F)}(x, Q^2)}{\frac{1}{2} f_{2}^{(P)}(x, Q^2)}$ as a function of $x$ for the values of $Q^2$, equal to 55 GeV$^2$, 65 GeV$^2$, 80 GeV$^2$, 105 GeV$^2$, 140 GeV$^2$ and 180 GeV$^2$. The experimental points are taken from Refs [5,4]. The solid lines show the theoretical calculations.

The manuscript was received 8 October 1985.